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reaction in which a photosensitive substance is broken down into *two* decomposition products. During dark adaptation of the retina these two decomposition products then unite to form again the sensitive substance from which they were produced. The actual course of dark adaptation depends on the change in concentration of these two reacting substances.

In all essentials, then, the mechanism underlying the initial phase of retinal sensitivity in dim light is the same as that which forms the basis of the initial process of photoreception in such forms as *Mya* and *Ciona*.

[1] W. Nagel, in Helmholtz' *Handbuch der physiologischen Optik*, **2**, 1911.

[2] S. Hecht, *Journal of General Physiology*, **1** and **2**, 1918–20.

A KINEMATICAL INTERPRETATION OF ELECTROMAGNETISM

By LEIGH PAGE

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Communicated by H. A. Bumstead. Read before the Academy, November 10, 1919

The concept of lines of force was introduced by Faraday as an aid in mapping out magnetic fields—these lines having everywhere the direction in which a small north pole would be urged by the field. Later it occurred to him that the characteristics of an electric field might be represented in similar manner by curves everywhere tangent to the electric intensity. Faraday limited his use of lines of force to static fields, and in cases where the field was produced by a number of charged particles, to the resultant field.

The electromagnetic theory of light, however, made it necessary to suppose that electric and magnetic lines of force in a wave front move with the velocity of light in a direction at right angles to their plane. Furthermore, the discovery of the electron made it natural to assume that when one of these small particles is in motion, its field is carried along with it. Wiechert and Stokes have explained X-rays as kinks in the lines of force emanating from such a particle—these kinks being propagated outward with the velocity of light whenever there is a change in the electron's state of motion.

The object of this paper is to show that the laws of electromagnetism may be explained exactly and in their entirety as kinematical relations between the moving elements which constitute lines of force, provided use is made of the space and time transformations of the principle of relativity in place of the more familiar but less philosophical Galilean transformations.[1] Each charged particle must be considered to have its own field, extending out to infinity in all directions, the resultant field at any point being a combination of those fields which extend to the point in question.

The electrodynamic equations are five in number: four of these specify the electric and magnetic fields in terms of the distribution of charge and current throughout space, and the fifth specifies the force on charged matter due to the strength of the electric and magnetic fields in which it is moving. First will be taken up the kinematical interpretation of the two equations —Coulomb's law and Ampere's law—which do not involve the space time transformations, and hence are not dependent on the principle of relativity.

In the representation to be employed, each charged particle e is to be considered as a source of moving elements projected outward uniformly in all directions with the velocity of light c. A line of force is a curve drawn through the moving elements which have been projected in a given direction. Perhaps a better picture of the method by which the field is produced may be obtained by imagining a very large number of guns to be stationed at e, pointing outward in all directions. Suppose each gun to be provided with a long, perfectly elastic cord on which shot are strung. Let these shot be fired from the gun at regular but very short intervals with the velocity of light. Then the shot constitute the moving elements emanating from the charge, and the elastic cord connecting them marks the line of force diverging in the particular direction under consideration.

The number of lines of force diverging from a charge de will be supposed to be very large, no matter how small de may be. For convenience in making numerical calculations, these lines will be grouped into bundles or tubes of M lines each, where M is a large number so chosen as to make the number of tubes N diverging from any charge equal numerically to the magnitude of the charge. This convention is tantamount to measuring electric charges in Heaviside-Lorentz rational units.

If the electric intensity $\mathbf{E}$ is defined as the number dN of tubes per unit cross section, the component of the intensity normal to the small surface $d\sigma$ is

$$\mathbf{E}_{d\sigma} \equiv \frac{dN}{d\sigma^2}\, d\sigma.$$

According to Gauss' theorem, the average divergence of $\mathbf{E}$ over a small volume $d\tau$ surrounded by the closed surface σ is given by

$$\begin{aligned}\nabla\cdot\mathbf{E}d\tau &= \int_\sigma \mathbf{E}\cdot d\sigma \\ &= \int_\sigma dN.\end{aligned}$$

Now the part of this sum due to charges outside the region $d\tau$ vanishes, while the part due to the charge de inside this region is equal to de itself. Hence, if ρ is the density of charge at the point in question,

$$\nabla\cdot\mathbf{E} = \rho. \qquad (1)$$

This is Coulomb's law.

To obtain Ampere's law, consider the increase in electric intensity at O (fig. 1) taking place in the time dt. During this time the lines of force at P will have moved up to O. If these lines are closer together at P than at O, $\mathbf{E}$ will change by an amount

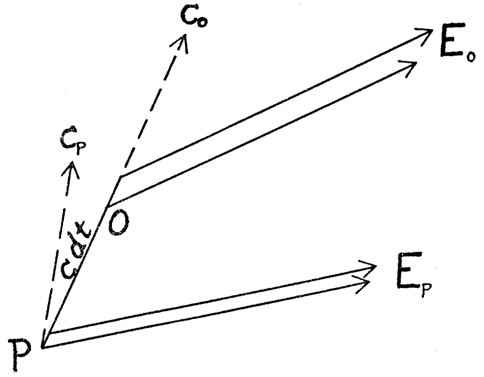

FIG. 1

$$-\mathbf{c}\cdot\nabla\mathbf{E}dt.$$

Furthermore $\mathbf{E}$ will suffer a change if the velocity $\mathbf{c}_P$ of the moving elements at P has a different direction from that at O, or if these lines twist as they move upward. The increase in $\mathbf{E}$ due to these two causes is

$$(-\mathbf{E}\nabla\cdot\mathbf{c} + \mathbf{E}\cdot\nabla\mathbf{c})dt.$$

Therefore, in all,

$$\begin{aligned}\dot{\mathbf{E}} &= -\mathbf{c}\cdot\nabla\mathbf{E} - \mathbf{E}\nabla\cdot\mathbf{c} + \mathbf{E}\cdot\nabla\mathbf{c}\\ &= -\mathbf{c}\nabla\cdot\mathbf{E} + \nabla\times(\mathbf{c}\times\mathbf{E}).\end{aligned}$$

So far, no account has been taken of the new lines of force produced by the emission of moving elements in the neighborhood of a charged particle. Let the dots in figure 2 represent sources emitting lines of force in the direction of the broken lines, and let these sources have a velocity $\mathbf{v}$ to the

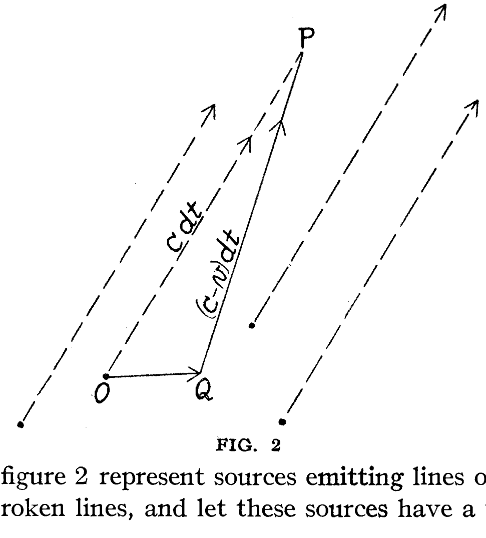

FIG. 2

right. Then the rate of change of the electric intensity at P due to the formation of new lines is

$$(\mathbf{c} - \mathbf{v})\nabla\cdot\mathbf{E}.$$

Adding this to the expression for $\dot{\mathbf{E}}$ obtained above, and using Coulomb's law to eliminate $\nabla\cdot\mathbf{E}$, it is seen that

$$\nabla \times (\mathbf{c} \times \mathbf{E}) = \dot{\mathbf{E}} + \rho\mathbf{v}.$$

The magnetic intensity $\mathbf{H}$ is defined by

$$\mathbf{H} \equiv \frac{1}{c}\mathbf{c} \times \mathbf{E}.$$

Therefore

$$\nabla \times \mathbf{H} = \frac{1}{c}(\dot{\mathbf{E}} + \rho\mathbf{v}). \qquad (2)$$

This is Ampere's law for the field due to a single charged particle. As it is linear in $\mathbf{E}$ and $\mathbf{H}$, the same law holds for the resultant of the simple fields due to any number of charged particles.

Next consider a charged particle e (fig. 3) at rest relative to the observer

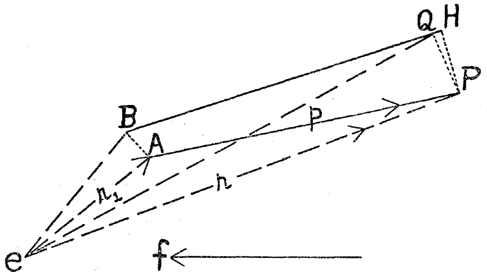

FIG. 3

but having an acceleration f to the left. Let eP be the path of a moving element emitted from the charge at the time zero, and eA that of a moving element belonging to the same line of force, but emitted at the time dt. As the velocity of the charge has changed in this time, these paths will make different angles with the acceleration. The line of force at P at time r/c will have the direction of AP, or $\mathbf{p}$. Evidently

$$\begin{aligned} \mathbf{r} &= \mathbf{c}t, \\ \mathbf{r}_1 &= \mathbf{c}t - \mathbf{c}dt + td\mathbf{c}, \\ \mathbf{p} &= \mathbf{c}dt - \frac{r}{c}d\mathbf{c}, \end{aligned}$$

and the electric intensity at P at the time r/c is given by

$$\begin{aligned} \mathbf{E} &= \frac{e}{4\pi r^2}\frac{\mathbf{p}}{p\cos QPH} \\ &= \frac{e}{4\pi r^2 c}\left\{\mathbf{c} - \frac{r}{c}\frac{d\mathbf{c}}{dt}\right\}. \end{aligned}$$

A short calculation gives

$$\frac{d\mathbf{c}}{dt} = -\frac{1}{c^2}(\mathbf{f} \times \mathbf{c}) \times \mathbf{c}.$$

Therefore,

$$\mathbf{E} = \frac{e}{4\pi r^2 c}\left\{\mathbf{c} + \frac{r}{c^3}(\mathbf{f} \times \mathbf{c}) \times \mathbf{c}\right\}. \quad (3)$$

So far no use has been made of the relativity transformations. In order to get the electric intensity due to a moving charge, recourse must be had to these transformations, resulting in the following expression,

$$\mathbf{E} = \frac{e}{4\pi r^2 k^2 c\left(1 - \frac{\mathbf{c}\cdot\mathbf{v}}{c^2}\right)^3}\left\{\mathbf{c} - \mathbf{v} + \frac{rk^2}{c^3}\{\mathbf{f} \times (\mathbf{c} - \mathbf{v})\} \times \mathbf{c}\right\}, \quad (4)$$

where

$$k \equiv \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}.$$

From the definition of magnetic intensity, it follows that

$$\mathbf{H} = \frac{e}{4\pi r^2 k^2 c\left(1 - \frac{\mathbf{c}\cdot\mathbf{v}}{c^2}\right)^3}\left\{-\frac{\mathbf{c} \times \mathbf{v}}{c} + \frac{rk^2}{c^4}\mathbf{c} \times (\{\mathbf{f} \times (\mathbf{c} - \mathbf{v})\} \times \mathbf{c})\right\}. \quad (5)$$

For fields specified by these expressions, it is easily shown that

$$\nabla \mathbf{H} = 0, \quad (6)$$

and

$$\nabla \times \mathbf{E} = -\frac{1}{c}\dot{\mathbf{H}}. \quad (7)$$

These are the two remaining field equations, the second being the mathematical expression of Faraday's law of current induction for the case where a current is induced by varying the magnetic flux through a stationary circuit. As these laws are linear in **E** and **H**, they hold as well for the resultant of the simple fields due to a number of charged particles as for each of these fields individually.

The last of the electromagnetic equations gives the force on a charge moving through electric and magnetic fields. Let unprimed letters in figure 4 refer to quantities as measured in a so-called stationary system, and primed letters to the same quantities as measured in a system moving to the right with velocity v. On account of the dependence of time on position required by the relativity transformations, the line of force *PE* in the stationary system has the different direction *PE'* in the moving system. If the dynamical equation for a charge e situated in the moving system is assumed to be

$$e\mathbf{E}' = \frac{d}{dt'}(m\mathbf{v}'),$$

as measured in the units of this system, the relativity transformations give

$$e\left(\mathbf{E} + \frac{1}{c}\,\mathbf{v} \times \mathbf{H}\right) = \frac{d}{dt}\left(\frac{m}{\sqrt{1 - \frac{v^2}{c^2}}}\,\mathbf{v}\right) \qquad (8)$$

in the stationary system. This is the fifth and last of the electromagnetic equations. If the relativity transformations had not been made use of in deriving it, the term involving the magnetic intensity would have been absent. But this term is the one that accounts for the current induced in a wire moving through a steady magnetic field, and for the torque in a coil carrying a current which is placed in such a field. Therefore, the action of every generator and every motor used by industry in this age of electricity is incontrovertible evidence of the truth of the principle of relativity.

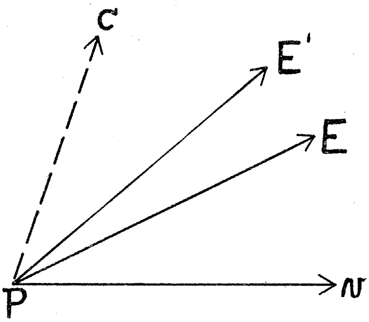

FIG. 4

In deriving expression (3) for the electric intensity due to a point charge, it has been tacitly assumed that the field does not rotate. If the sources of moving elements constituting the charge—the guns in the analogy previously used—are supposed to rotate with angular velocity ω, the rate of change of $\mathbf{c}$ at the time r/c becomes

$$\frac{d\mathbf{c}}{dt} = -\frac{1}{c^2}\left[(\mathbf{f} \times \mathbf{c}) \times \mathbf{c} + \left\{(\omega \times \mathbf{c}) \times \mathbf{c}\right\} \times \mathbf{c}\right],$$

for a charge which is at rest relative to the observer at the time zero. This introduces into the expression (3) for $\mathbf{E}$ the additional term

$$\frac{e}{4\pi r^2 c}\left\{\frac{r}{c^3}\left\{(\omega \times \mathbf{c}) \times \mathbf{c}\right\} \times \mathbf{c}\right\},$$

and $\mathbf{E}$ and $\mathbf{H}$ for a moving charge become

$$\mathbf{E} = \frac{e}{4\pi r^2 k^2 c\left(1 - \frac{\mathbf{c}\cdot\mathbf{v}}{c^2}\right)^3}\left\{\mathbf{c} - \mathbf{v} + \frac{rk^2}{c^3}\left\{\mathbf{f} \times (\mathbf{c} - \mathbf{v})\right\} \times \mathbf{c} + \frac{r}{c^3}\left(\left\{\omega \times (\mathbf{c} - \mathbf{v})\right\} \times \mathbf{c}\right) \times \mathbf{c}\right\}, \quad (9)$$

$$\mathbf{H} = \frac{e}{4\pi r^2 k^2 c\left(1 - \frac{\mathbf{c}\cdot\mathbf{v}}{c^2}\right)^3}\left\{-\frac{\mathbf{c} \times \mathbf{v}}{c} + \frac{rk^2}{c^4}\mathbf{c} \times \left(\left\{\mathbf{f} \times (\mathbf{c} - \mathbf{v})\right\} \times \mathbf{c}\right) + \frac{r}{c^2}\left\{\omega \times (\mathbf{c} - \mathbf{v})\right\} \times \mathbf{c}\right\}. \quad (10)$$

The field equations (1) and (2) representing Coulomb's law and Ampere's law are unchanged for this more general type of field, but (5) and (6) become

$$\nabla\cdot\mathbf{H} = \epsilon, \quad (11)$$

$$\nabla \times \mathbf{E} = -\frac{1}{c}(\dot{\mathbf{H}} + \epsilon\mathbf{u}), \quad (12)$$

where the density of magnetic charge ϵ is

$$\epsilon \equiv \frac{e}{2\pi r^2 k^2 c^2}\,\frac{\frac{\omega\cdot\mathbf{c}}{k^2} - \omega\cdot\mathbf{v}\left(1 - \frac{\mathbf{c}\cdot\mathbf{v}}{c^2}\right)}{\left(1 - \frac{\mathbf{c}\cdot\mathbf{v}}{c^2}\right)^4}, \quad (13)$$

and its velocity u is in general equal to the velocity c of light. The energy equation for fields of this character takes the form

$$\frac{d}{dt}\left\{\frac{1}{2}(E^2 + H^2)\right\} + c\nabla\cdot(\mathbf{E} \times \mathbf{H}) + \mathbf{E}\cdot\mathbf{v}\rho + \mathbf{H}\cdot\mathbf{u}\epsilon = 0, \quad (14)$$

and the energy radiated by a charge e due to rotation of the field is

$$R = \frac{e^2\omega}{3c}$$

for each revolution. In terms of the frequency ν this expression has the form

$$R = \frac{2\pi e^2}{3c}\nu.$$

If the value of the electronic charge is substituted for e, to wit

$$e = 4.77\,(10)^{-10}\sqrt{4\pi}$$

Heaviside-Lorentz units, it is found that

$$R = \frac{1}{32}\,6.4(10)^{-27}\,\nu \text{ erg sec.}$$

$$= \frac{1}{32}h\nu,$$

where h is Planck's constant. This numerical coincidence suggests that quantum phenomena may be in some way connected with rotating electric fields, and that

$$h \equiv \frac{64\pi e^2}{3c}.$$

Summary.—The equations of electrodynamics are shown to be simple kinematical relations between the moving elements which constitute lines of force. If the sources of these moving elements are supposed to rotate, free magnetic charges are produced, and the four equations of the electromagnetic field assume a symmetrical form. The energy radiated due to the rotation is found to amount almost exactly to $^1/_{32}\, h\nu$ for each revolution.

[1] The reader who is interested in following through in more detail the analysis involved in deriving these relations is referred to the author's paper on "Relativity and the Ether," *Amer. J. Sci., New Haven,* **38,** 1914 (169).

THE LAWS OF ELASTICO-VISCOUS FLOW. II

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Communicated January 19, 1920

In a paper of Harold Jeffreys entitled "The Viscosity of the Earth,"[1] the author makes use of a formula which combines the laws of Larmor and of Maxwell.

$$n\left(S + \tau_2 \frac{ds}{dt}\right) = F + \frac{1}{\tau_1}\int F dt.$$

The integral implies a permanent set which as the author indicates would be inconsistent with the "accepted theories of tidal friction and variation of latitude. Hence τ_1 must be practically infinite." The formula is thus reduced to the expression $F = n_1(S + \tau_2\, ds/dt)$.

Experiments made on a great variety of materials show, however, that this expression must be seriously modified to represent the facts.

Thus it has been shown[2] that the displacement produced by a stress P is given by the expression[3] $S = C_1 P e^{h_1 P} + C_2 P e^{h_2 P}(1 - e^{-\alpha\sqrt{t}}) + C_3 P e^{h_3} t^{\rho}$. The last term produces permanent set, so for the present may be omitted. Putting

$$C_1 = \frac{1}{n_1} \text{ and } C_2 = \frac{1}{n_2} \quad \alpha = \frac{1}{\tau} \quad \Theta = t^{1/2}$$

this becomes

$$S = \frac{P}{n_1}\left(A_1 + \frac{n_1 A_2}{n_2}\left(1 - e^{-\frac{\Theta}{\tau}}\right)\right)$$

whence

$$n_1\left(S - \tau\frac{ds}{d\Theta}\right) = P\left(A_1 + \frac{n_1}{n_2}A_2\right).$$